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One-sided capability indices C_{PU} and C_{PL}: decision making with sample information

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Abstract *Process capability indices have been used in the manufacturing industry to provide quantitative measures on process potential and performance. The formulae for these indices are easy to understand and straightforward to apply. But, since sample data must be collected in order to calculate these indices, a great degree of uncertainty may be introduced into capability assessments due to sampling errors. Currently, most practitioners simply look at the value of the index calculated from the sample data and then make a conclusion on whether their processes meet the capability requirement. This approach is not reliable since sampling errors are ignored. Procedures for two-sided capability indices,* C_p, C_{pk}, *and* C_{pm} *have been developed to assist practitioners to determine whether their processes meet the capability requirement based on sample information. In this paper, we first obtain unbiased estimators of* C_{PU} *and* C_{PL}*. We then develop a procedure similar to those of* C_p, C_{pk}, *and* C_{pm}, *for the one-sided capability indices* C_{PU} *and* C_{PL}*. Practitioners can use the procedure to test whether their processes meet the capability requirement.*

Introduction

Several capability indices including C_p, C_{PU}, C_{PL}, C_{pk}, and C_{pm}, have been used in the manufacturing industry to provide common quantitative measures on process potential and performance (see Boyles (1991), Chan *et al.* (1988), Kane (1986), and Pearn *et al.* (1992)). Those indices are defined in the following:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma},$$

$$C_{\mathrm{PU}} = \frac{\mathrm{USL} - \mu}{3\sigma},$$

$$C_{\mathrm{PL}} = \frac{\mu - \mathrm{LSL}}{3\sigma},$$

$$C_{\mathrm{pk}} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\},$$

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$$C_{\mathrm{pm}} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, σ is the process standard deviation (overall process variation), and T is the target value.

While C_p, C_{pk}, and C_{pm} are appropriate measures for processes with two-sided specifications (which require both upper and lower specification limits USL and LSL), C_{PU} and C_{PL} have been designed particularly for processes with one-sided specifications (which require only the upper or the lower specification limit). The formulae for these indices are easy to understand and straightforward to apply. In practice, however, sample data must be collected in order to calculate these indices. Therefore, a great degree of uncertainty may be introduced into capability assessments due to sampling errors. Currently, most practitioners simply look at the value of the estimators calculated from the sample data, then make a conclusion on whether their processes meet the preset capability (quality) requirement. This approach is highly unreliable since sampling errors are ignored.

Taking the sampling errors into account, Cheng (1992, 1994), Pearn and Chen (1996a, b) have developed simple but practical procedures for C_p, C_{pk}, and C_{pm}, to assist the practitioners to determine whether their processes meet the capability requirement. But, no procedure has been developed for the one-sided capability indices C_{PU} and C_{PL}. In this paper, we develop a similar procedure for the one-sided capability indices C_{PU} and C_{PL}. Practitioners can use the procedure to test whether their processes meet the capability requirement.

Estimations of C_{PU} and C_{PL}

To estimate the indices C_{PU} and C_{PL}, we consider the natural estimators, which are defined as the following:

$$\hat{C}_{\mathrm{PU}} = \frac{\mathrm{USL} - \overline{X}}{3S},$$

$$\hat{C}_{\mathrm{PL}} = \frac{\overline{X} - \mathrm{LSL}}{3S},$$

where $\bar{X} = \sum_{\mathrm{i}=1}^{n} X_i/n,\ S = [(n-1)^{-1}\sum_{\mathrm{i}=1}^{n}(X_i - \bar{X})^2]^{1/2}$ are conventional estimators of μ and σ, which may be obtained from a process that is demonstrably stable (in-control). Chou and Owen (1989) investigated the estimators $\hat{C}_{\mathrm{PU}}$ and $\hat{C}_{\mathrm{PL}}$, and showed that under normality assumption, the estimators $\hat{C}_{\mathrm{PU}}$ and $\hat{C}_{\mathrm{PL}}$ are distributed as $ct_{n-1}(\delta)$, where $c = (3\sqrt{n})^{-1}$, and $t_{n-1}(\delta)$ is a non-central t distribution with $n-1$ degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n}\ C_{\mathrm{PU}}$ and $\delta = 3\sqrt{n}\ C_{\mathrm{PL}}$ respectively.

Both estimators are biased. But, we can show that by adding the well-known correction factor $b_{n-1} = [2/(n-1)]^{1/2}\,\Gamma[(n-1)/2]\Gamma\,[(n-2)/2]$ to $\hat{C}_{\mathrm{PU}}$ and $\hat{C}_{\mathrm{PL}}$, we obtain unbiased estimators $b_{n-1}\,\hat{C}_{PU}$ and $b_{n-1}\,\hat{C}_{PL}$ which we denote as $\tilde{C}_{PU}$ and $\tilde{C}_{PL}$. Thus, we have $E(\tilde{C}_{PU}) = C_{PU}$, and $E(\tilde{C}_{PL}) = C_{PL}$. Since $b_{n-1} < 1$ ($n > 2$), then $\mathrm{Var}(\tilde{C}_{PU}) < \mathrm{Var}(\hat{C}_{PU})$ and $\mathrm{Var}(\tilde{C}_{PL}) < \mathrm{Var}(\hat{C}_{PL})$. Calculations needed to obtain these results are straightforward (see Appendix 1). Further, since both estimators depend only on the sufficient and complete statistics $(\bar{X}, S^2)$ of (μ, σ^2), $\tilde{C}_{PU}$ and $\tilde{C}_{PL}$ are uniformly minimum variance unbiased estimators (UMVUE) of C_{PU} and C_{PL} respectively. The r-th moment (about zero) and the variance of $\tilde{C}_{PU}$ can be obtained as in the following (see Appendix 1), where $Z = \sqrt{n}\,(\mathrm{USL} - \bar{X})/\sigma$. It is easy to verify that $E(\tilde{C}_{PU}) = C_{PU}$. The results of the r-th moment, the expected value, and the variance of the other estimator $\tilde{C}_{PL}$ are the same.

$$E[\tilde{C}_{PU}]^r = \frac{(\Gamma[(n-1)/2])^{r-1}\Gamma[(n-1-r)/2]}{(3\sqrt{n})^r(\Gamma[(n-2)/2])^r}\mathrm{E}(\mathrm{Z})^{\mathrm{r}},$$

$$\mathrm{Var}[\tilde{C}_{PU}] = \left\{\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2} - 1\right\}[C_{PU}]^2 + \frac{1}{9n}\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2}.$$

Capability requirement

The statistical properties of the two estimator $\tilde{C}_{PU}$, and $\tilde{C}_{PL}$ are exactly the same. For convenience of presentation, we let C_I be either C_{PU} or C_{PL}. In current practice, a process is called "inadequate" if $C_I < 1.00$; it indicates that the process is not adequate with respect to the production tolerances (specifications), either process variation (σ^2) needs to be reduced or process mean (μ) needs to be shifted closer to the target value T. A process is called capable if $1.00 \leq C_I < 1.33$; it indicates that caution needs to be taken regarding process distribution, some process control is required. A process is called satisfactory if $1.33 \leq C_I < 1.50$; it indicates that process quality is satisfactory, material substitution may be allowed, and no stringent quality control is required. A process is called excellent if $1.50 \leq C_I < 2.00$; it indicates that process quality exceeds "satisfactory". A process satisfies Motorola's capability requirement (see Harry (1988)) if $C_{pk} \geq 1.50$ and $C_p \geq 2.00$. Thus, for processes with one-sided specifications, Motorola's capability requirement is equivalent to "excellent". Finally, a process is called "super" if $C_I \geq 2.00$. Table I summarizes the five conditions and the corresponding C_I values.

Quality condition	C_I values
Inadequate	$C_I < 1.00$
Capable	$1.00 \leq C_I < 1.33$
Satisfactory	$1.33 \leq C_I < 1.50$
Excellent	$1.50 \leq C_I < 2.00$
Super	$2.00 \leq C_I$

Table I. The five quality conditions

Testing hypothesis

To test whether a given process meets the preset capability requirement and runs under the desired quality condition, we can consider the following statistical testing hypothesis. A process meets the capability (quality) requirement if $C_I > C$ (a preset known constant), and fails to meet the capability requirement if $C_I \leq C$:

H0. $C_I \leq C$.
H1. $C_I > C$.

In the following, we calculate the p-value (rejection probability), critical value, and power of the test. Suppose the observed value of the statistic $\tilde{C}_I = W$, then we can calculate those values as the following, where $\delta = 3\sqrt{n}\,C$:

$$\begin{aligned}
p\text{-value} &= P\left\{\tilde{C}_I \geq W | C_I \leq C\right\} \\
&= P\left\{b_{n-1}\hat{C}_I \geq W | C_I \leq C\right\} \\
&= P\left\{\hat{C}_I \geq \frac{W}{b_{n-1}} | C_I \leq C\right\} \\
&= P\left\{3\sqrt{n}\hat{C}_I \geq \frac{3\sqrt{n}W}{b_{n-1}} | C_I \leq C\right\} \\
&= P\left\{t_{n-1}(\delta) \geq \frac{3\sqrt{n}W}{b_{n-1}} | C_I \leq C\right\}.
\end{aligned}$$

The critical value, C_0, is determined by

$$\begin{aligned}
&P\left\{\tilde{C}_I \geq C_0 | C_I = C\right\} = \alpha \\
&P\left\{b_{n-1}\hat{C}_I \geq C_0 | C_I = C\right\} = \alpha \\
&P\left\{\hat{C}_I \geq \frac{C_0}{b_{n-1}} | C_I = C\right\} = \alpha \\
&P\left\{3\sqrt{n}\hat{C}_I \geq \frac{3\sqrt{n}C_0}{b_{n-1}} | C_I = C\right\} = \alpha \\
&P\left\{t_{n=1}(\delta) \geq \frac{3\sqrt{n}C_0}{b_{n-1}} | C_I = C\right\} = \alpha
\end{aligned}$$

where $\delta = 3\sqrt{n}\,C$. Hence, we have

$$\frac{3\sqrt{n}C_0}{b_{n-1}} = t_{n-1}, \alpha(\delta),$$

where t_{n-1}, α (δ) is the upper α-th quantile of $t_{n-1}(\delta)$ distribution, or

$$C_0 = \frac{b_{n-1}}{3\sqrt{n}} t_{n-1}, \alpha(\delta).$$

The power of the test can be computed as the following, where $\delta = 3\sqrt{n}\,C_I$.

$$\begin{aligned}\pi(C_I) &= P\left\{\tilde{C}_I > C_0 | C_I\right\} \\ &= P\left\{b_{n-1}\hat{C}_I > C_0 | C_I\right\} \\ &= P\left\{\hat{C}_I > \frac{C_0}{b_{n-1}} | C_I\right\} \\ &= P\left\{3\sqrt{n}\hat{C}_I > \frac{3\sqrt{n}C_0}{b_{n-1}} | C_\mathrm{I}\right\} \\ &= P\left\{t_{n-1}(\delta) > \frac{3\sqrt{n}C_0}{b_{n-1}}\right\}.\end{aligned}$$

The procedure

Tables AI-AIV (see Appendix 2) listed in Table II display critical values C_0 for commonly used C = 1.00, 1.33, 1.50, and 2.00, and powers π at various C_I values with sample sizes n = 10(5)250, and α-risk = 0.01, 0.025, 0.05. To determine if the process meets the capability (quality) requirement, we first determine C, and the α-risk. Then, we calculate the value of the estimator $\tilde{C}_I$ from the sample. From the appropriate table, we find the critical value C_0 based on α-risk, C, and the sample size n. If the estimated value $\tilde{C}_I$ is greater than the critical value C_0 then we conclude that the process meets the capability requirement. Otherwise, we do not have sufficient information to conclude that the process meets the preset capability requirement.

C	Table	Sample size n	α	C_I values
1.00	AI, part A	10(5)250	0.010	1.00(0.10)1.80
	AI, part B	10(5)250	0.025	1.00(0.10)1.80
	AI, part C	10(5)250	0.050	1.00(0.10)1.80
1.33	AII, part A	10(5)250	0.010	1.33(0.10)2.13
	AII, part B	10(5)250	0.025	1.33(0.10)2.13
	AII, part C	10(5)250	0.050	1.33(0.10)2.13
1.50	AIII, part A	10(5)250	0.010	1.50(0.10)2.30
	AIII, part B	10(5)250	0.025	1.50(0.10)2.30
	AIII, part C	10(5)250	0.050	1.50(0.10)2.30
2.00	AIV, part A	10(5)250	0.010	2.00(0.10)2.80
	AIV, part B	10(5)250	0.025	2.00(0.10)2.80
	AIV, part C	10(5)250	0.050	2.00(0.10)2.80

Table II. Guidelines for the table selections

The procedure is as follows:

- *Step 1*. Determine the value of C (normally chosen from Table I), the desired quality condition, and the α-risk (normally set to 0.01, 0.025, or 0.05), the chance of incorrectly concluding a bad process (quality does not meet the capability requirement) or good (quality meets the preset capability requirement).
- *Step 2*. Calculate the value of the estimator, $\tilde{C}_I$, from the sample.
- *Step 3*. Check the appropriate table listed in Table II and find the corresponding C_0 based on α, C, and the sample size n.
- *Step 4*. Conclude that the process meets the capability requirement if $\tilde{C}_I$ is greater than C_0. Otherwise, we do not have enough information to conclude that the process meets the capability requirement.

To simplify the calculations of the estimators $\tilde{C}_I$, we have provided values of the correction factor b_{n-1} for various sample sizes $n = 10(5)250$ (see Table III). In Figures 1(a)-(c), 2(a)-(c), 3a-c, and 4(a)-(c), we plot the power curves ($\pi(C_I)$ versus C_I value) for the four quality conditions with C set to the minimal values, C = 1.00, 1.33. 1.50, 2.00, α-risk = 0.01,0.025, 0.05, and sample sizes $n = 10(5)250$.

For short run applications, we also generate the critical values C_0 for C = 1.00, 1.33, 1.50, and 2.00, with α-risk = 0.05, 0.025, and 0.01 for small sample sizes n = 3(1)30, as displayed in Table IV. Thus, for a short run with LSL = 50 and n = 8, with sample mean $\bar{X}$ = 53.18, and sample standard deviation $S = 0.61$, we can compute $b_{n-1} = [2/(8-1)]^{1/2}\,\Gamma[(8-1)/2]/\Gamma[(8-2)/2] = 0.869$, and the estimator $\tilde{C}_{PL} = b_{n-1}\,(\bar{X} - LSL)/(3S) = 1.512$. Assume the α-risk is 0.05, if we just look at the point estimation, $\tilde{C}_{PL}$ = 1.512 > 1.33, we may conclude that the process meets the capability requirement "satisfactory". This is certainly not a reliable conclusion. Checking Table IV based on $C = 1.33$, α = 0.05, and n = 8 obtaining C_0 = 2.154. Since $\tilde{C}_{PL}$ = 1.512 is less than the critical value C_0 = 2.154, we would reject the conclusion that the process meets the capability requirement "satisfactory" since the information supporting the conclusion is insufficient.

An application example

Consider the following example taken from a company located in Taiwan, the Republic of China, manufacturing and supplying nylon strings used for deep-sea

Table III. b_{n-1} values for various sample sizes n

n	b_{n-1}	n	b_{n-1}	n	b_{n-1}	n	b_{n-1}	n	b_{n-1}	n	b_{n-1}	n	b_{n-1}
10	0.914	45	0.983	80	0.990	115	0.993	150	0.995	185	0.996	220	0.997
15	0.945	50	0.985	85	0.991	120	0.994	155	0.995	190	0.996	225	0.997
20	0.960	55	0.986	90	0.992	125	0.994	160	0.995	195	0.996	230	0.997
25	0.968	60	0.987	95	0.992	130	0.994	165	0.995	200	0.996	235	0.997
30	0.974	65	0.988	100	0.992	135	0.994	170	0.996	205	0.996	240	0.997
35	0.978	70	0.989	105	0.993	140	0.995	175	0.996	210	0.996	245	0.997
40	0.981	75	0.990	110	0.993	145	0.995	180	0.996	215	0.996	250	0.997

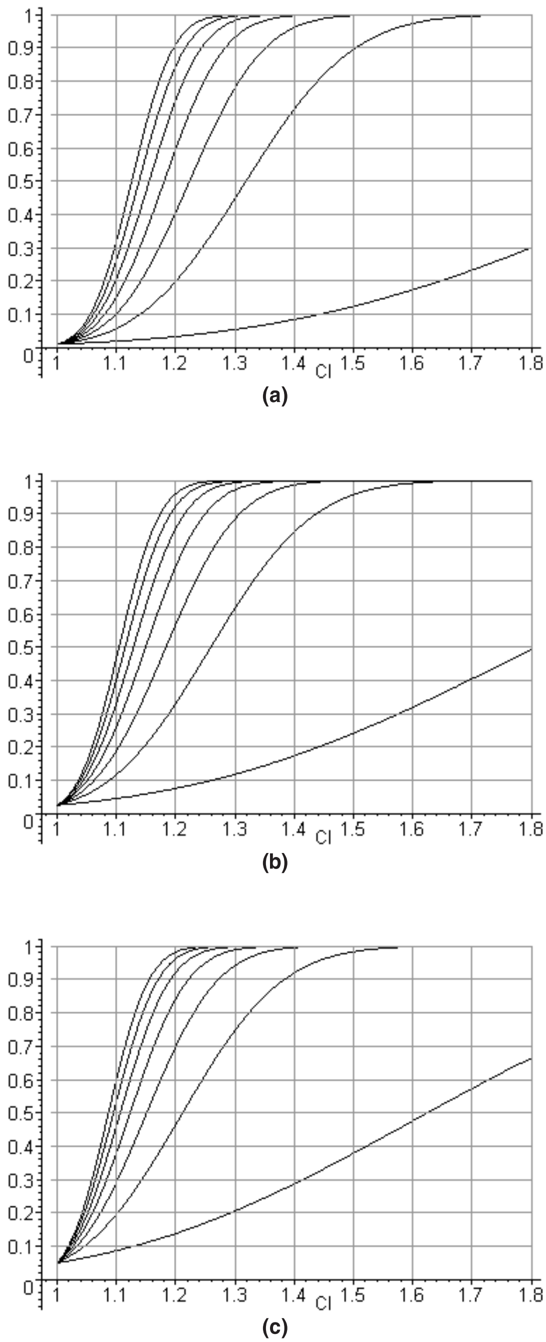

Figure 1.
Power curves for $C = 1.00$ and $n = 10(40)250$ (from bottom to top in plot). with (a) $\alpha = 0.01$; (b) $\alpha = 0.025$; and (c) $\alpha = 0.05$

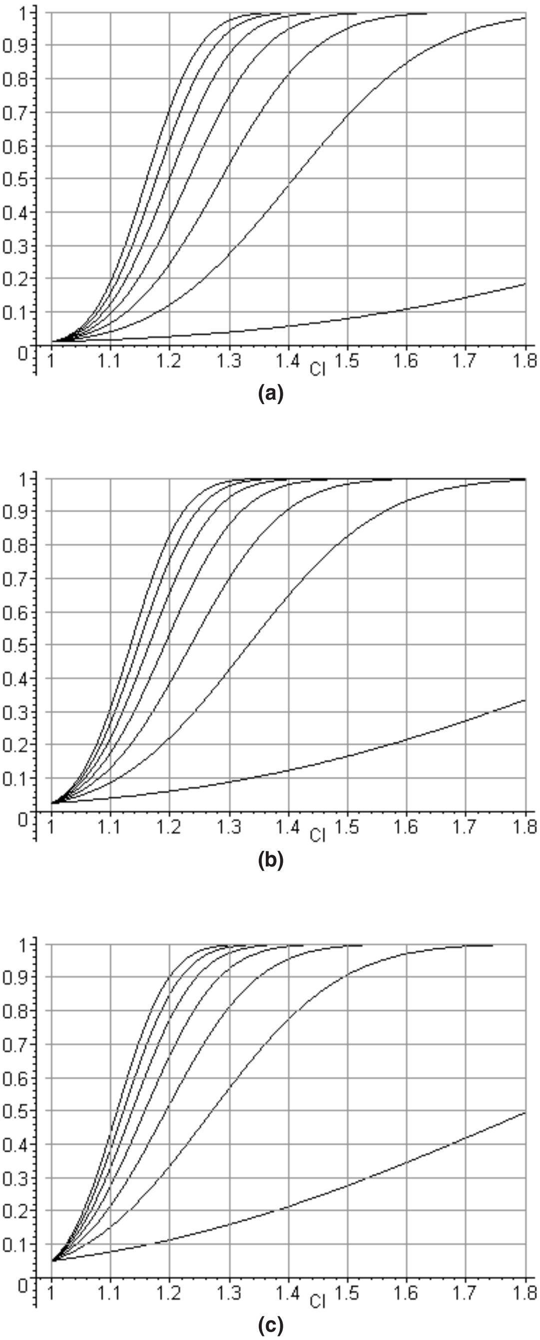

Figure 2.
Power curves for $C = 1.33$ and $n = 10(40)250$ (from bottom to top in plot) with: (a) $\alpha = 0.01$; (b) $\alpha = 0.025$; and (c) $\alpha = 0.05$

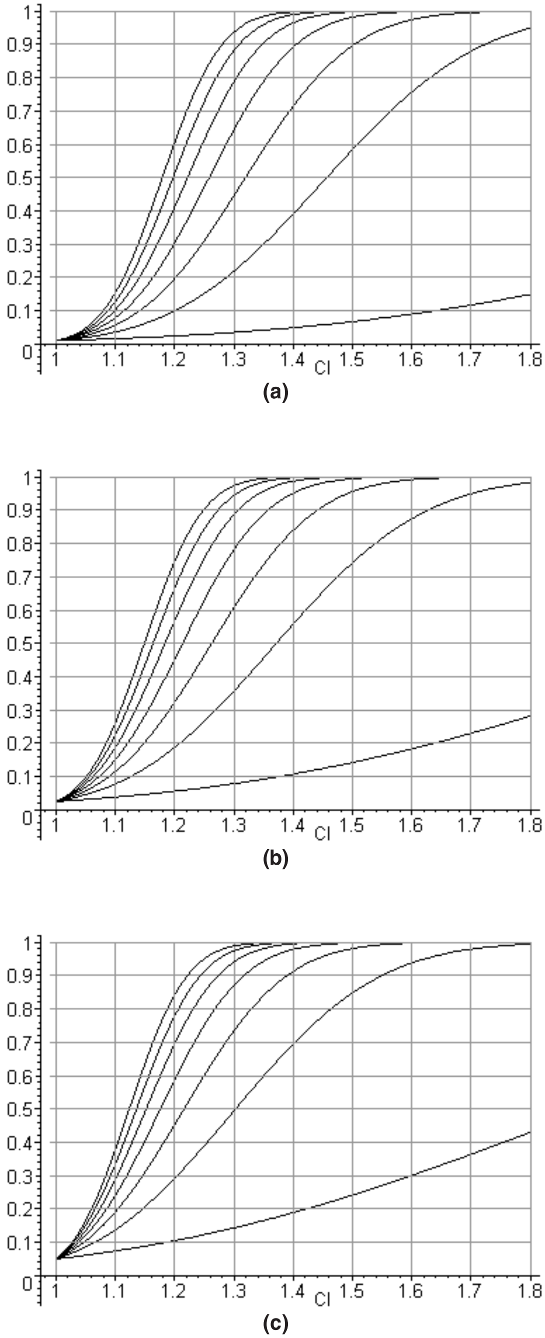

Figure 3.
Power curves for $C = 1.50$ and $n = 10(40)250$ (from bottom to top in plot) with: (a) $\alpha = 0.01$; (b) $\alpha = 0.025$; and (c) $\alpha = 0.05$

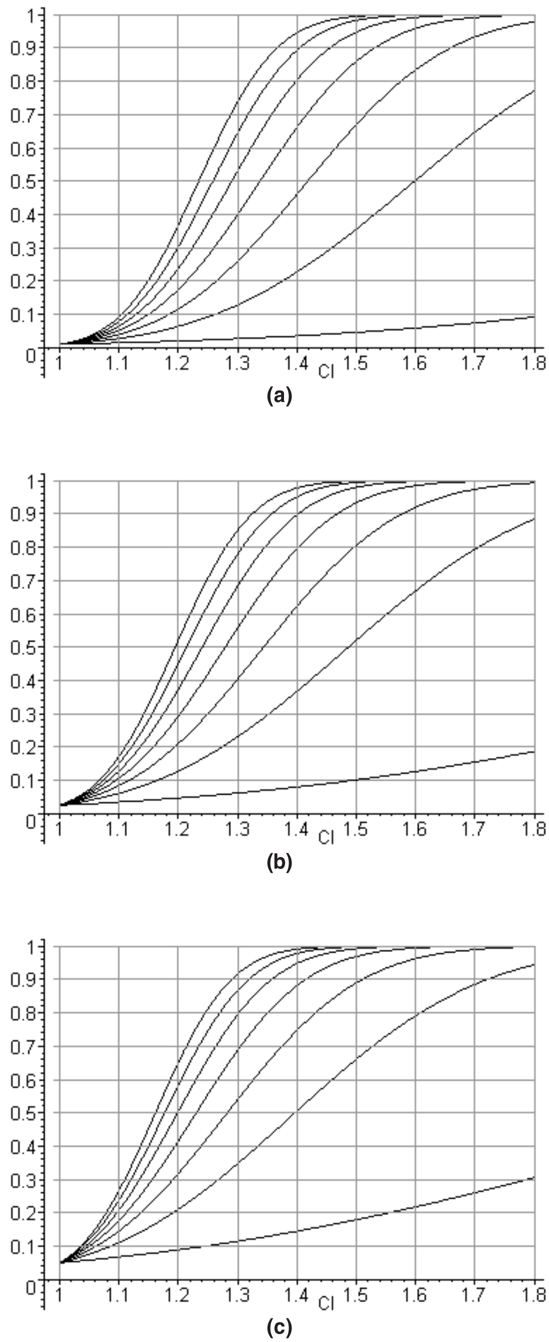

Figure 4.
Power curves for $C = 2.00$ and $n = 10(40)250$ (from bottom to top in plot) with: (a) $\alpha = 0.01$; (b) $\alpha = 0.025$; and (c) $\alpha = 0.05$

$n \mid \alpha$	C = 1.00			C = 1.33			C = 1.50			C = 2.00		
	0.01	0.025	0.05	0.01	0.025	0.05	0.01	0.025	0.05	0.01	0.025	0.05
3	5.890	3.709	2.603	7.774	4.896	3.438	8.749	5.511	3.870	11.620	7.323	5.144
4	3.799	2.768	2.164	4.999	3.644	2.851	5.621	4.098	3.207	7.457	5.438	4.257
5	3.013	2.355	1.940	3.960	3.098	2.554	4.450	3.482	2.872	5.900	4.619	3.811
6	2.603	2.120	1.801	3.418	2.787	2.371	3.841	3.133	2.665	5.090	4.154	3.536
7	2.350	1.968	1.707	3.085	2.586	2.246	3.466	2.907	2.525	4.594	3.854	3.350
8	2.178	1.860	1.637	2.859	2.444	2.154	3.212	2.747	2.422	4.257	3.642	3.214
9	2.052	1.779	1.584	2.695	2.338	2.084	3.028	2.628	2.343	4.012	3.484	3.109
10	1.957	1.715	1.541	2.569	2.255	2.028	2.887	2.535	2.281	3.826	3.361	3.026
11	1.881	1.664	1.506	2.470	2.188	1.983	2.775	2.460	2.230	3.678	3.262	2.959
12	1.819	1.622	1.477	2.389	2.133	1.944	2.685	2.398	2.187	3.558	3.180	2.902
13	1.768	1.586	1.452	2.322	2.087	1.912	2.609	2.346	2.150	3.458	3.111	2.854
14	1.724	1.556	1.430	2.265	2.047	1.884	2.545	2.301	2.119	3.374	3.053	2.812
15	1.686	1.529	1.411	2.216	2.012	1.859	2.490	2.263	2.091	3.302	3.002	2.776
16	1.654	1.506	1.394	2.173	1.982	1.837	2.443	2.229	2.067	3.239	2.957	2.744
17	1.625	1.486	1.379	2.136	1.955	1.818	2.401	2.199	2.045	3.183	2.917	2.715
18	1.599	1.467	1.366	2.103	1.932	1.801	2.363	2.172	2.026	3.134	2.882	2.690
19	1.576	1.451	1.354	2.073	1.910	1.785	2.330	2.148	2.008	3.090	2.850	2.666
20	1.556	1.436	1.343	2.046	1.891	1.771	2.300	2.126	1.992	3.050	2.821	2.645
21	1.537	1.422	1.333	2.022	1.873	1.757	2.273	2.106	1.977	3.015	2.795	2.626
22	1.520	1.409	1.323	2.000	1.856	1.745	2.248	2.088	1.964	2.982	2.771	2.608
23	1.504	1.398	1.315	1.979	1.841	1.734	2.225	2.071	1.951	2.952	2.749	2.592
24	1.490	1.387	1.307	1.961	1.828	1.724	2.204	2.056	1.940	2.924	2.729	2.577
25	1.477	1.377	1.299	1.943	1.815	1.714	2.185	2.041	1.929	2.899	2.710	2.562
26	1.464	1.368	1.293	1.927	1.803	1.705	2.167	2.028	1.919	2.875	2.692	2.549
27	1.453	1.359	1.286	1.912	1.792	1.697	2.150	2.016	1.910	2.853	2.676	2.537
28	1.442	1.351	1.280	1.898	1.781	1.689	2.135	2.004	1.901	2.833	2.661	2.525
29	1.432	1.344	1.274	1.885	1.772	1.682	2.120	1.993	1.893	2.813	2.646	2.515
30	1.422	1.337	1.269	1.873	1.762	1.675	2.106	1.983	1.885	2.795	2.633	2.504

Table IV. Critical values C_0 for $C = 1.00, 1.33, 1.50, 2.00$, with α-risk = 0.05, 0.025, 0.01, and small sample sizes ($n \leq 30$)

fishing rods. The production specifications for a particular model of ultra-strong nylon string has a lower specification limit LSL = 50lb of pull force. The capability requirement for this particular model of nylon string was defined as "satisfactory" ($C_{PL} > 1.33$). A total of 100 observations were collected which are displayed in Table V.

The company wants to determine whether the manufacturing process meets the capability requirement "satisfactory". We first calculate the sample mean

54.29	53.88	53.46	52.92	53.86	53.28	53.27	52.94	53.86	53.14
53.10	52.39	53.91	52.17	54.46	53.06	53.12	52.73	53.56	53.90
53.82	53.43	53.76	53.74	53.84	53.66	53.51	53.45	53.89	53.33
54.73	53.91	53.33	53.25	53.78	53.00	52.02	53.20	52.51	53.85
54.36	53.53	54.60	54.87	52.67	53.71	53.06	53.44	52.95	53.65
54.04	54.25	53.08	53.58	53.06	52.25	54.17	52.40	52.74	54.08
53.83	53.71	53.50	53.72	52.85	53.38	52.83	53.21	53.04	54.79
53.29	52.58	53.29	53.07	53.27	53.50	53.19	53.24	53.22	54.20
53.02	53.90	52.34	53.26	53.81	52.32	53.87	54.30	53.29	53.06
53.32	53.06	53.75	52.31	53.91	54.39	54.06	53.88	54.49	53.84

Table V. Sample data with 100 observations

$\bar{X}$ = 53.44, and the sample standard deviation S = 0.60. Checking the value of the correction factor b_{n-1} from Table III, we obtain b_{n-1} = 0.992. Therefore, we may calculate the estimator $\tilde{C}_{PL} = b_{n-1} (\bar{X} - \text{LSL})/(3S) = 1.895$. Assume the α-risk is 0.05, then we find the critical value C_0 = 1.506 from Table AII, part B based on C = 1.33, α = 0.05, and sample size n = 100. Since $\tilde{C}_{PL}$ = 1.895 is greater than the critical value C_0 = 1.506 in this case, we therefore conclude that the process meets the capability requirement "satisfactory".

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Appendix 1

Theorem

If the process follows the normal distribution, then

$$E\left[\tilde{C}_{PU}\right]^r = \frac{(\Gamma[(n-1)/2])^{r-1}\Gamma[(n-1-r)/2]}{(3\sqrt{n})^r(\Gamma[(n-2)/2])^r}E(Z)^r, Z = \frac{\sqrt{n}(\text{USL}-\bar{X})}{\sigma}$$

$$\text{Var}\left[\tilde{C}_{PU}\right] = \left\{\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2} - 1\right\}[C_{PU}]^2 + \frac{1}{9\text{n}}\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^2}$$

$\tilde{C}_{PU}$ is an UMVUE of C_{PU}.

Proof

$$\tilde{C}_{PU} = b_{n-1}\hat{C}_{PU} = b_{n-1}\left[\frac{\text{USL}-\bar{X}}{3\text{S}}\right]$$

$$= \left[\frac{b_{n-1}}{3}\right]\left[\frac{n-1}{n}\right]^{1/2}\left[\frac{(n-1)S^2}{\sigma^2}\right]^{-1/2}\left[\frac{\sqrt{n}(\text{USL}-\bar{X})}{\sigma}\right]$$

$$= \left[\frac{b_{n-1}}{3}\right]\left[\frac{n-1}{n}\right]^{1/2}(K)^{-1/2}(Z), \text{where } K = (n-1)S^2/\sigma^2$$

is distributed as χ^2_{n-1}, and $Z = \sqrt{n}\ (\text{USL}-\bar{X})/\sigma$ is distributed as $N(3\sqrt{n}\ C_{PU},\ 1)$. Since $\bar{X}$ and S^2 are mutually independent, so are Z and K. Hence,

$$
\begin{aligned}
\mathrm{E}(\tilde{C}_{\mathrm{PU}})^{\mathrm{r}} &= \left[\frac{b_{n-1}}{3}\right]^{r}\left[\frac{n-1}{n}\right]^{\mathrm{r}/2} E(K)^{-r/2}E(Z)^{r} \\
&= \left[\frac{b_{n-1}}{3}\right]^{r}\left[\frac{n-1}{n}\right]^{r/2}\frac{\Gamma[(n-1-r)/2]}{2^{r/2}\Gamma[(n-1)/2]}E(Z)^{r} \\
&= \frac{(\Gamma[(n-1)/2])^{r-1}\Gamma[(n-1-r)/2]}{(3\sqrt{n})^{r}(\Gamma[(n-2)/2])^{r}}E(Z)^{r}.
\end{aligned}
$$

We note that $E(Z) = 3\sqrt{n}C_{\mathrm{PU}}$, and $E(Z)^2 = 9n(C_{\mathrm{PU}})^2 + 1$. Therefore,

$$
\begin{aligned}
E(\tilde{C}_{\mathrm{PU}}) &= \left[\frac{b_{n-1}}{3}\right]\left[\frac{n-1}{n}\right]^{1/2}\frac{\Gamma[(n-2)/2]}{2^{1/2}\Gamma[(n-1)/2]}3n^{1/2}C_{\mathrm{PU}} = C_{\mathrm{PU}}, \\
E(\tilde{C}_{\mathrm{PU}})^{2} &= \frac{(b_{n-1})^{2}}{9}\left[\frac{n-1}{n}\right]\frac{\Gamma[(n-3)/2]}{2\Gamma[(n-1)/2]}\left[9n(C_{\mathrm{PU}})^{2}+1\right], \\
\mathrm{Var}(\tilde{C}_{\mathrm{PU}}) &= E(\tilde{C}_{\mathrm{PU}})^{2} - \left[E(\tilde{C}_{\mathrm{PU}})\right]^{2} \\
&= \left\{\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^{2}} - 1\right\}[C_{\mathrm{PU}}]^{2} + \frac{1}{9n}\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{(\Gamma[(n-2)/2])^{2}}.
\end{aligned}
$$

Since $\tilde{C}_{\mathrm{PU}}$ is unbiased estimator of C_{PU} and depends only on the sufficient and complete statistics $(\bar{X}, S^2)$, it follows that $\tilde{C}_{\mathrm{PU}}$ is a UMVUE of C_{PU}.

Appendix 2

n	C_0	C_I values 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
A. Critical values C_0 for C = 1.00 and powers π at various C_I values for n = 10(5)250 with α = 0.01										
10	1.957	0.010	0.019	0.033	0.054	0.084	0.124	0.173	0.233	0.300
15	1.686	0.010	0.024	0.049	0.091	0.154	0.237	0.339	0.452	0.566
20	1.556	0.010	0.028	0.067	0.134	0.235	0.366	0.511	0.652	0.773
25	1.477	0.010	0.033	0.086	0.182	0.323	0.493	0.661	0.799	0.896
30	1.422	0.010	0.038	0.106	0.233	0.413	0.609	0.778	0.894	0.957
35	1.383	0.010	0.042	0.128	0.287	0.499	0.708	0.862	0.947	0.984
40	1.352	0.010	0.047	0.151	0.341	0.579	0.788	0.917	0.975	0.994
45	1.327	0.010	0.052	0.174	0.395	0.652	0.849	0.952	0.989	0.998
50	1.307	0.010	0.057	0.199	0.448	0.716	0.896	0.973	0.995	0.999
55	1.290	0.010	0.063	0.224	0.500	0.771	0.929	0.986	0.998	1.000
60	1.275	0.010	0.068	0.250	0.549	0.817	0.953	0.992	0.999	1.000
65	1.262	0.010	0.073	0.276	0.595	0.856	0.969	0.996	1.000	1.000
70	1.251	0.010	0.079	0.302	0.639	0.887	0.980	0.998	1.000	1.000
75	1.241	0.010	0.085	0.328	0.679	0.912	0.987	0.999	1.000	1.000
80	1.233	0.010	0.090	0.354	0.716	0.933	0.992	1.000	1.000	1.000
85	1.225	0.010	0.096	0.381	0.750	0.949	0.995	1.000	1.000	1.000
90	1.217	0.010	0.102	0.406	0.780	0.961	0.997	1.000	1.000	1.000
95	1.211	0.010	0.108	0.432	0.808	0.971	0.998	1.000	1.000	1.000
100	1.205	0.010	0.114	0.457	0.833	0.978	0.999	1.000	1.000	1.000
105	1.199	0.010	0.120	0.482	0.855	0.984	0.999	1.000	1.000	1.000
110	1.194	0.010	0.127	0.506	0.874	0.988	1.000	1.000	1.000	1.000
115	1.189	0.010	0.133	0.530	0.892	0.991	1.000	1.000	1.000	1.000
120	1.185	0.010	0.139	0.553	0.907	0.994	1.000	1.000	1.000	1.000
125	1.181	0.010	0.146	0.575	0.920	0.995	1.000	1.000	1.000	1.000
130	1.177	0.010	0.152	0.597	0.932	0.997	1.000	1.000	1.000	1.000
135	1.173	0.010	0.159	0.618	0.942	0.998	1.000	1.000	1.000	1.000
140	1.170	0.010	0.166	0.638	0.950	0.998	1.000	1.000	1.000	1.000
145	1.166	0.010	0.172	0.657	0.958	0.999	1.000	1.000	1.000	1.000
150	1.163	0.010	0.179	0.676	0.964	0.999	1.000	1.000	1.000	1.000
155	1.160	0.010	0.186	0.694	0.970	0.999	1.000	1.000	1.000	1.000
160	1.157	0.010	0.193	0.711	0.975	1.000	1.000	1.000	1.000	1.000
165	1.155	0.010	0.200	0.728	0.979	1.000	1.000	1.000	1.000	1.000
170	1.152	0.010	0.207	0.744	0.982	1.000	1.000	1.000	1.000	1.000
175	1.150	0.010	0.213	0.759	0.985	1.000	1.000	1.000	1.000	1.000
180	1.148	0.010	0.220	0.773	0.987	1.000	1.000	1.000	1.000	1.000
185	1.145	0.010	0.227	0.787	0.990	1.000	1.000	1.000	1.000	1.000
190	1.143	0.010	0.235	0.800	0.991	1.000	1.000	1.000	1.000	1.000
195	1.141	0.010	0.242	0.813	0.993	1.000	1.000	1.000	1.000	1.000
200	1.139	0.010	0.249	0.824	0.994	1.000	1.000	1.000	1.000	1.000
205	1.138	0.010	0.256	0.836	0.995	1.000	1.000	1.000	1.000	1.000
210	1.136	0.010	0.263	0.846	0.996	1.000	1.000	1.000	1.000	1.000
215	1.134	0.010	0.270	0.856	0.997	1.000	1.000	1.000	1.000	1.000
220	1.132	0.010	0.277	0.866	0.997	1.000	1.000	1.000	1.000	1.000
225	1.131	0.010	0.284	0.874	0.998	1.000	1.000	1.000	1.000	1.000
230	1.129	0.010	0.292	0.883	0.998	1.000	1.000	1.000	1.000	1.000
235	1.128	0.010	0.299	0.891	0.998	1.000	1.000	1.000	1.000	1.000
240	1.126	0.010	0.306	0.898	0.999	1.000	1.000	1.000	1.000	1.000
245	1.125	0.010	0.313	0.905	0.999	1.000	1.000	1.000	1.000	1.000
250	1.124	0.010	0.320	0.912	0.999	1.000	1.000	1.000	1.000	1.000

(Continued)

Table AI.

		C_I values								
n	C_0	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
B. Critical values C_0 for C = 1.00 and powers π at various C_I values for n = 10(5)250 with α = 0.025										
10	1.715	0.025	0.045	0.076	0.118	0.174	0.241	0.319	0.405	0.493
15	1.529	0.025	0.055	0.105	0.181	0.281	0.399	0.525	0.646	0.752
20	1.436	0.025	0.064	0.136	0.247	0.390	0.547	0.694	0.814	0.898
25	1.377	0.025	0.073	0.168	0.315	0.495	0.673	0.816	0.911	0.963
30	1.337	0.025	0.082	0.200	0.381	0.589	0.771	0.895	0.960	0.988
35	1.306	0.025	0.091	0.233	0.445	0.671	0.845	0.942	0.983	0.996
40	1.283	0.025	0.099	0.265	0.507	0.741	0.897	0.970	0.993	0.999
45	1.264	0.025	0.108	0.298	0.564	0.799	0.934	0.984	0.997	1.000
50	1.248	0.025	0.117	0.331	0.617	0.846	0.958	0.992	0.999	1.000
55	1.235	0.025	0.126	0.363	0.665	0.883	0.974	0.996	1.000	1.000
60	1.223	0.025	0.135	0.394	0.709	0.912	0.984	0.998	1.000	1.000
65	1.213	0.025	0.144	0.426	0.748	0.934	0.990	0.999	1.000	1.000
70	1.205	0.025	0.153	0.456	0.783	0.952	0.994	1.000	1.000	1.000
75	1.197	0.025	0.162	0.485	0.814	0.965	0.997	1.000	1.000	1.000
80	1.190	0.025	0.171	0.514	0.841	0.974	0.998	1.000	1.000	1.000
85	1.184	0.025	0.180	0.541	0.865	0.981	0.999	1.000	1.000	1.000
90	1.178	0.025	0.189	0.568	0.885	0.987	0.999	1.000	1.000	1.000
95	1.173	0.025	0.198	0.593	0.903	0.990	1.000	1.000	1.000	1.000
100	1.168	0.025	0.207	0.618	0.918	0.993	1.000	1.000	1.000	1.000
105	1.163	0.025	0.216	0.641	0.931	0.995	1.000	1.000	1.000	1.000
110	1.159	0.025	0.225	0.664	0.943	0.997	1.000	1.000	1.000	1.000
115	1.155	0.025	0.234	0.685	0.952	0.998	1.000	1.000	1.000	1.000
120	1.152	0.025	0.243	0.706	0.960	0.998	1.000	1.000	1.000	1.000
125	1.148	0.025	0.252	0.725	0.967	0.999	1.000	1.000	1.000	1.000
130	1.145	0.025	0.261	0.743	0.972	0.999	1.000	1.000	1.000	1.000
135	1.142	0.025	0.271	0.760	0.977	0.999	1.000	1.000	1.000	1.000
140	1.140	0.025	0.280	0.777	0.981	1.000	1.000	1.000	1.000	1.000
145	1.137	0.025	0.289	0.792	0.985	1.000	1.000	1.000	1.000	1.000
150	1.135	0.025	0.298	0.807	0.987	1.000	1.000	1.000	1.000	1.000
155	1.132	0.025	0.307	0.820	0.990	1.000	1.000	1.000	1.000	1.000
160	1.130	0.025	0.315	0.833	0.991	1.000	1.000	1.000	1.000	1.000
165	1.128	0.025	0.324	0.845	0.993	1.000	1.000	1.000	1.000	1.000
170	1.126	0.025	0.333	0.856	0.994	1.000	1.000	1.000	1.000	1.000
175	1.124	0.025	0.342	0.867	0.995	1.000	1.000	1.000	1.000	1.000
180	1.122	0.025	0.351	0.877	0.996	1.000	1.000	1.000	1.000	1.000
185	1.120	0.025	0.360	0.886	0.997	1.000	1.000	1.000	1.000	1.000
190	1.118	0.025	0.368	0.895	0.998	1.000	1.000	1.000	1.000	1.000
195	1.117	0.025	0.377	0.903	0.998	1.000	1.000	1.000	1.000	1.000
200	1.115	0.025	0.385	0.910	0.998	1.000	1.000	1.000	1.000	1.000
205	1.114	0.025	0.394	0.917	0.999	1.000	1.000	1.000	1.000	1.000
210	1.112	0.025	0.403	0.924	0.999	1.000	1.000	1.000	1.000	1.000
215	1.111	0.025	0.411	0.930	0.999	1.000	1.000	1.000	1.000	1.000
220	1.110	0.025	0.419	0.935	0.999	1.000	1.000	1.000	1.000	1.000
225	1.108	0.025	0.428	0.940	0.999	1.000	1.000	1.000	1.000	1.000
230	1.107	0.025	0.436	0.945	1.000	1.000	1.000	1.000	1.000	1.000
235	1.106	0.025	0.444	0.949	1.000	1.000	1.000	1.000	1.000	1.000
240	1.105	0.025	0.452	0.954	1.000	1.000	1.000	1.000	1.000	1.000
245	1.103	0.025	0.460	0.957	1.000	1.000	1.000	1.000	1.000	1.000
250	1.102	0.025	0.468	0.961	1.000	1.000	1.000	1.000	1.000	1.000

(Continued)

Table AI.

Table AI.

n	C_0	C_I values 1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80
C. Critical values C_0 for C = 1.00 and powers π at various C_I values for n = 10(5)250 with $\alpha = 0.05$										
10	1.541	0.050	0.087	0.138	0.206	0.287	0.379	0.476	0.573	0.664
15	1.411	0.050	0.102	0.183	0.292	0.421	0.557	0.684	0.790	0.871
20	1.343	0.050	0.117	0.227	0.376	0.542	0.698	0.823	0.908	0.957
25	1.299	0.050	0.131	0.270	0.454	0.645	0.802	0.906	0.963	0.987
30	1.269	0.050	0.144	0.311	0.526	0.729	0.874	0.953	0.986	0.997
35	1.246	0.050	0.158	0.352	0.591	0.797	0.922	0.977	0.995	0.999
40	1.228	0.050	0.170	0.391	0.650	0.849	0.953	0.989	0.998	1.000
45	1.213	0.050	0.183	0.429	0.702	0.890	0.972	0.995	0.999	1.000
50	1.201	0.050	0.196	0.465	0.747	0.920	0.983	0.998	1.000	1.000
55	1.190	0.050	0.208	0.500	0.787	0.942	0.990	0.999	1.000	1.000
60	1.181	0.050	0.220	0.533	0.821	0.959	0.995	1.000	1.000	1.000
65	1.174	0.050	0.233	0.565	0.850	0.971	0.997	1.000	1.000	1.000
70	1.167	0.050	0.245	0.595	0.875	0.980	0.998	1.000	1.000	1.000
75	1.161	0.050	0.256	0.623	0.897	0.986	0.999	1.000	1.000	1.000
80	1.155	0.050	0.268	0.650	0.914	0.990	0.999	1.000	1.000	1.000
85	1.150	0.050	0.280	0.675	0.929	0.993	1.000	1.000	1.000	1.000
90	1.145	0.050	0.292	0.699	0.942	0.995	1.000	1.000	1.000	1.000
95	1.141	0.050	0.303	0.722	0.952	0.997	1.000	1.000	1.000	1.000
100	1.137	0.050	0.314	0.743	0.961	0.998	1.000	1.000	1.000	1.000
105	1.134	0.050	0.326	0.762	0.968	0.999	1.000	1.000	1.000	1.000
110	1.131	0.050	0.337	0.781	0.974	0.999	1.000	1.000	1.000	1.000
115	1.127	0.050	0.348	0.798	0.979	0.999	1.000	1.000	1.000	1.000
120	1.125	0.050	0.359	0.814	0.983	1.000	1.000	1.000	1.000	1.000
125	1.122	0.050	0.369	0.829	0.986	1.000	1.000	1.000	1.000	1.000
130	1.119	0.050	0.380	0.842	0.989	1.000	1.000	1.000	1.000	1.000
135	1.117	0.050	0.391	0.855	0.991	1.000	1.000	1.000	1.000	1.000
140	1.115	0.050	0.401	0.867	0.993	1.000	1.000	1.000	1.000	1.000
145	1.113	0.050	0.411	0.878	0.994	1.000	1.000	1.000	1.000	1.000
150	1.111	0.050	0.421	0.888	0.995	1.000	1.000	1.000	1.000	1.000
155	1.109	0.050	0.432	0.898	0.996	1.000	1.000	1.000	1.000	1.000
160	1.107	0.050	0.442	0.906	0.997	1.000	1.000	1.000	1.000	1.000
165	1.105	0.050	0.451	0.914	0.998	1.000	1.000	1.000	1.000	1.000
170	1.104	0.050	0.461	0.922	0.998	1.000	1.000	1.000	1.000	1.000
175	1.102	0.050	0.471	0.928	0.999	1.000	1.000	1.000	1.000	1.000
180	1.100	0.050	0.480	0.935	0.999	1.000	1.000	1.000	1.000	1.000
185	1.099	0.050	0.490	0.940	0.999	1.000	1.000	1.000	1.000	1.000
190	1.098	0.050	0.499	0.946	0.999	1.000	1.000	1.000	1.000	1.000
195	1.096	0.050	0.508	0.950	0.999	1.000	1.000	1.000	1.000	1.000
200	1.095	0.050	0.517	0.955	1.000	1.000	1.000	1.000	1.000	1.000
205	1.094	0.050	0.526	0.959	1.000	1.000	1.000	1.000	1.000	1.000
210	1.093	0.050	0.535	0.963	1.000	1.000	1.000	1.000	1.000	1.000
215	1.092	0.050	0.543	0.966	1.000	1.000	1.000	1.000	1.000	1.000
220	1.090	0.050	0.552	0.969	1.000	1.000	1.000	1.000	1.000	1.000
225	1.089	0.050	0.560	0.972	1.000	1.000	1.000	1.000	1.000	1.000
230	1.088	0.050	0.568	0.974	1.000	1.000	1.000	1.000	1.000	1.000
235	1.087	0.050	0.576	0.977	1.000	1.000	1.000	1.000	1.000	1.000
240	1.086	0.050	0.585	0.979	1.000	1.000	1.000	1.000	1.000	1.000
245	1.086	0.050	0.592	0.981	1.000	1.000	1.000	1.000	1.000	1.000
250	1.085	0.050	0.600	0.983	1.000	1.000	1.000	1.000	1.000	1.000

n	C_0	C_I values 1.33	1.43	1.53	1.63	1.73	1.83	1.93	2.03	2.13
A. Critical values C_0 *for* C = *1.33 and powers* π *at various* C_I *values for* n = *10(5)250 with* α = *0.01*										
10	2.569	0.010	0.017	0.026	0.039	0.057	0.080	0.109	0.143	0.184
15	2.216	0.010	0.020	0.036	0.061	0.097	0.145	0.206	0.279	0.360
20	2.046	0.010	0.023	0.046	0.085	0.143	0.222	0.318	0.427	0.540
25	1.943	0.010	0.026	0.058	0.113	0.196	0.305	0.434	0.568	0.693
30	1.873	0.010	0.029	0.069	0.142	0.251	0.391	0.544	0.688	0.807
35	1.822	0.010	0.032	0.082	0.173	0.309	0.474	0.642	0.784	0.885
40	1.782	0.010	0.035	0.095	0.206	0.367	0.553	0.725	0.855	0.935
45	1.750	0.010	0.038	0.108	0.239	0.425	0.625	0.794	0.906	0.964
50	1.724	0.010	0.041	0.122	0.274	0.481	0.690	0.848	0.940	0.981
55	1.702	0.010	0.044	0.136	0.309	0.534	0.746	0.890	0.963	0.990
60	1.683	0.010	0.047	0.151	0.344	0.585	0.794	0.922	0.978	0.995
65	1.666	0.010	0.050	0.166	0.378	0.632	0.835	0.945	0.987	0.998
70	1.652	0.010	0.054	0.182	0.413	0.676	0.869	0.962	0.992	0.999
75	1.639	0.010	0.057	0.198	0.447	0.716	0.897	0.974	0.996	0.999
80	1.628	0.010	0.060	0.214	0.480	0.752	0.919	0.982	0.997	1.000
85	1.618	0.010	0.064	0.230	0.512	0.784	0.937	0.988	0.999	1.000
90	1.608	0.010	0.067	0.246	0.543	0.814	0.952	0.992	0.999	1.000
95	1.600	0.010	0.070	0.263	0.574	0.839	0.963	0.995	1.000	1.000
100	1.592	0.010	0.074	0.279	0.602	0.862	0.972	0.997	1.000	1.000
105	1.585	0.010	0.077	0.296	0.630	0.882	0.979	0.998	1.000	1.000
110	1.578	0.010	0.081	0.312	0.657	0.900	0.984	0.999	1.000	1.000
115	1.572	0.010	0.085	0.329	0.682	0.915	0.988	0.999	1.000	1.000
120	1.567	0.010	0.088	0.346	0.705	0.928	0.991	0.999	1.000	1.000
125	1.561	0.010	0.092	0.362	0.728	0.939	0.993	1.000	1.000	1.000
130	1.556	0.010	0.096	0.379	0.749	0.949	0.995	1.000	1.000	1.000
135	1.551	0.010	0.099	0.395	0.769	0.957	0.996	1.000	1.000	1.000
140	1.547	0.010	0.103	0.411	0.787	0.964	0.997	1.000	1.000	1.000
145	1.543	0.010	0.107	0.428	0.805	0.970	0.998	1.000	1.000	1.000
150	1.539	0.010	0.111	0.444	0.821	0.975	0.999	1.000	1.000	1.000
155	1.535	0.010	0.114	0.459	0.836	0.979	0.999	1.000	1.000	1.000
160	1.532	0.010	0.118	0.475	0.850	0.983	0.999	1.000	1.000	1.000
165	1.528	0.010	0.122	0.490	0.863	0.986	0.999	1.000	1.000	1.000
170	1.525	0.010	0.126	0.506	0.875	0.988	1.000	1.000	1.000	1.000
175	1.522	0.010	0.130	0.521	0.886	0.990	1.000	1.000	1.000	1.000
180	1.519	0.010	0.134	0.535	0.897	0.992	1.000	1.000	1.000	1.000
185	1.516	0.010	0.138	0.550	0.906	0.994	1.000	1.000	1.000	1.000
190	1.513	0.010	0.142	0.564	0.915	0.995	1.000	1.000	1.000	1.000
195	1.511	0.010	0.146	0.578	0.923	0.996	1.000	1.000	1.000	1.000
200	1.508	0.010	0.150	0.592	0.930	0.997	1.000	1.000	1.000	1.000
205	1.506	0.010	0.155	0.605	0.937	0.997	1.000	1.000	1.000	1.000
210	1.504	0.010	0.159	0.618	0.943	0.998	1.000	1.000	1.000	1.000
215	1.501	0.010	0.163	0.631	0.948	0.998	1.000	1.000	1.000	1.000
220	1.499	0.010	0.167	0.643	0.953	0.998	1.000	1.000	1.000	1.000
225	1.497	0.010	0.171	0.656	0.958	0.999	1.000	1.000	1.000	1.000
230	1.495	0.010	0.175	0.668	0.962	0.999	1.000	1.000	1.000	1.000
235	1.493	0.010	0.180	0.679	0.966	0.999	1.000	1.000	1.000	1.000
240	1.492	0.010	0.184	0.691	0.969	0.999	1.000	1.000	1.000	1.000
245	1.490	0.010	0.188	0.702	0.972	0.999	1.000	1.000	1.000	1.000
250	1.488	0.010	0.193	0.712	0.975	1.000	1.000	1.000	1.000	1.000

(Continued)

Table AII.

		C_I values								
n	C_0	1.33	1.43	1.53	1.63	1.73	1.83	1.93	2.03	2.13
B. Critical values C_0 for C = 1.33 and powers π at various C_I values for n = 10(5)250 with α = 0.025										
10	2.255	0.025	0.040	0.061	0.088	0.123	0.166	0.216	0.272	0.335
15	2.012	0.025	0.047	0.080	0.128	0.190	0.268	0.357	0.453	0.550
20	1.891	0.025	0.053	0.099	0.169	0.262	0.373	0.494	0.613	0.721
25	1.815	0.025	0.059	0.119	0.212	0.333	0.474	0.615	0.740	0.839
30	1.762	0.025	0.065	0.139	0.255	0.404	0.566	0.715	0.833	0.913
35	1.723	0.025	0.070	0.160	0.298	0.472	0.648	0.795	0.896	0.955
40	1.693	0.025	0.076	0.180	0.342	0.536	0.719	0.856	0.938	0.977
45	1.668	0.025	0.082	0.201	0.384	0.595	0.778	0.900	0.963	0.989
50	1.648	0.025	0.087	0.222	0.426	0.649	0.827	0.932	0.979	0.995
55	1.631	0.025	0.093	0.243	0.466	0.697	0.866	0.955	0.988	0.998
60	1.616	0.025	0.099	0.264	0.505	0.740	0.898	0.970	0.994	0.999
65	1.604	0.025	0.104	0.284	0.542	0.779	0.922	0.980	0.997	1.000
70	1.592	0.025	0.110	0.305	0.577	0.812	0.942	0.987	0.998	1.000
75	1.582	0.025	0.115	0.326	0.611	0.842	0.956	0.992	0.999	1.000
80	1.573	0.025	0.121	0.346	0.642	0.867	0.968	0.995	0.999	1.000
85	1.565	0.025	0.127	0.367	0.672	0.889	0.976	0.997	1.000	1.000
90	1.558	0.025	0.132	0.387	0.700	0.907	0.983	0.998	1.000	1.000
95	1.551	0.025	0.138	0.407	0.726	0.923	0.987	0.999	1.000	1.000
100	1.545	0.025	0.144	0.426	0.750	0.936	0.991	0.999	1.000	1.000
105	1.539	0.025	0.149	0.445	0.773	0.947	0.993	1.000	1.000	1.000
110	1.534	0.025	0.155	0.464	0.794	0.957	0.995	1.000	1.000	1.000
115	1.529	0.025	0.161	0.483	0.813	0.964	0.997	1.000	1.000	1.000
120	1.524	0.025	0.166	0.501	0.830	0.971	0.998	1.000	1.000	1.000
125	1.520	0.025	0.172	0.519	0.847	0.976	0.998	1.000	1.000	1.000
130	1.516	0.025	0.178	0.536	0.862	0.981	0.999	1.000	1.000	1.000
135	1.512	0.025	0.183	0.553	0.875	0.984	0.999	1.000	1.000	1.000
140	1.509	0.025	0.189	0.570	0.888	0.987	0.999	1.000	1.000	1.000
145	1.505	0.025	0.195	0.586	0.899	0.990	1.000	1.000	1.000	1.000
150	1.502	0.025	0.200	0.602	0.909	0.992	1.000	1.000	1.000	1.000
155	1.499	0.025	0.206	0.617	0.919	0.993	1.000	1.000	1.000	1.000
160	1.496	0.025	0.212	0.632	0.927	0.995	1.000	1.000	1.000	1.000
165	1.493	0.025	0.217	0.646	0.935	0.996	1.000	1.000	1.000	1.000
170	1.491	0.025	0.223	0.660	0.942	0.997	1.000	1.000	1.000	1.000
175	1.488	0.025	0.229	0.674	0.948	0.997	1.000	1.000	1.000	1.000
180	1.486	0.025	0.235	0.687	0.953	0.998	1.000	1.000	1.000	1.000
185	1.484	0.025	0.240	0.700	0.958	0.998	1.000	1.000	1.000	1.000
190	1.481	0.025	0.246	0.712	0.963	0.999	1.000	1.000	1.000	1.000
195	1.479	0.025	0.252	0.724	0.967	0.999	1.000	1.000	1.000	1.000
200	1.477	0.025	0.257	0.736	0.971	0.999	1.000	1.000	1.000	1.000
205	1.475	0.025	0.263	0.747	0.974	0.999	1.000	1.000	1.000	1.000
210	1.474	0.025	0.269	0.758	0.977	0.999	1.000	1.000	1.000	1.000
215	1.472	0.025	0.274	0.769	0.980	1.000	1.000	1.000	1.000	1.000
220	1.470	0.025	0.280	0.779	0.982	1.000	1.000	1.000	1.000	1.000
225	1.468	0.025	0.286	0.788	0.984	1.000	1.000	1.000	1.000	1.000
230	1.467	0.025	0.291	0.798	0.986	1.000	1.000	1.000	1.000	1.000
235	1.465	0.025	0.297	0.807	0.987	1.000	1.000	1.000	1.000	1.000
240	1.464	0.025	0.302	0.815	0.989	1.000	1.000	1.000	1.000	1.000
245	1.462	0.025	0.308	0.824	0.990	1.000	1.000	1.000	1.000	1.000
250	1.461	0.025	0.314	0.832	0.991	1.000	1.000	1.000	1.000	1.000

Table AII.

(Continued)

n	C_0	C_I values 1.33	1.43	1.53	1.63	1.73	1.83	1.93	2.03	2.13
C. Critical values C_0 *for* $C = 1.33$ *and powers* π *at various* C_I *values for* $n = 10(5)250$ *with* $\alpha = 0.05$										
10	2.028	0.050	0.077	0.113	0.158	0.213	0.276	0.345	0.419	0.494
15	1.859	0.050	0.088	0.144	0.216	0.305	0.405	0.510	0.613	0.706
20	1.771	0.050	0.099	0.173	0.273	0.393	0.523	0.647	0.757	0.843
25	1.714	0.050	0.108	0.201	0.328	0.475	0.624	0.754	0.854	0.921
30	1.675	0.050	0.117	0.229	0.381	0.550	0.709	0.833	0.916	0.963
35	1.645	0.050	0.126	0.257	0.431	0.618	0.778	0.889	0.953	0.983
40	1.622	0.050	0.135	0.284	0.479	0.677	0.832	0.928	0.974	0.992
45	1.603	0.050	0.144	0.310	0.525	0.729	0.875	0.954	0.986	0.997
50	1.587	0.050	0.152	0.336	0.567	0.774	0.908	0.971	0.993	0.999
55	1.574	0.050	0.160	0.361	0.607	0.813	0.932	0.982	0.996	0.999
60	1.562	0.050	0.168	0.386	0.644	0.845	0.951	0.989	0.998	1.000
65	1.552	0.050	0.177	0.410	0.678	0.873	0.965	0.993	0.999	1.000
70	1.543	0.050	0.185	0.434	0.710	0.896	0.975	0.996	1.000	1.000
75	1.535	0.050	0.193	0.457	0.738	0.915	0.982	0.998	1.000	1.000
80	1.528	0.050	0.200	0.479	0.765	0.931	0.987	0.999	1.000	1.000
85	1.522	0.050	0.208	0.501	0.789	0.944	0.991	0.999	1.000	1.000
90	1.516	0.050	0.216	0.522	0.811	0.955	0.994	0.999	1.000	1.000
95	1.511	0.050	0.224	0.543	0.831	0.964	0.996	1.000	1.000	1.000
100	1.506	0.050	0.231	0.562	0.849	0.971	0.997	1.000	1.000	1.000
105	1.501	0.050	0.239	0.582	0.866	0.977	0.998	1.000	1.000	1.000
110	1.497	0.050	0.246	0.600	0.880	0.981	0.999	1.000	1.000	1.000
115	1.493	0.050	0.254	0.618	0.894	0.985	0.999	1.000	1.000	1.000
120	1.489	0.050	0.261	0.635	0.906	0.988	0.999	1.000	1.000	1.000
125	1.486	0.050	0.269	0.652	0.916	0.991	1.000	1.000	1.000	1.000
130	1.483	0.050	0.276	0.668	0.926	0.993	1.000	1.000	1.000	1.000
135	1.480	0.050	0.283	0.683	0.934	0.994	1.000	1.000	1.000	1.000
140	1.477	0.050	0.290	0.698	0.942	0.995	1.000	1.000	1.000	1.000
145	1.474	0.050	0.298	0.712	0.949	0.996	1.000	1.000	1.000	1.000
150	1.471	0.050	0.305	0.726	0.955	0.997	1.000	1.000	1.000	1.000
155	1.469	0.050	0.312	0.739	0.960	0.998	1.000	1.000	1.000	1.000
160	1.467	0.050	0.319	0.752	0.965	0.998	1.000	1.000	1.000	1.000
165	1.465	0.050	0.326	0.764	0.969	0.999	1.000	1.000	1.000	1.000
170	1.462	0.050	0.333	0.776	0.973	0.999	1.000	1.000	1.000	1.000
175	1.460	0.050	0.340	0.787	0.976	0.999	1.000	1.000	1.000	1.000
180	1.458	0.050	0.347	0.797	0.979	0.999	1.000	1.000	1.000	1.000
185	1.457	0.050	0.353	0.807	0.982	1.000	1.000	1.000	1.000	1.000
190	1.455	0.050	0.360	0.817	0.984	1.000	1.000	1.000	1.000	1.000
195	1.453	0.050	0.367	0.826	0.986	1.000	1.000	1.000	1.000	1.000
200	1.452	0.050	0.374	0.835	0.988	1.000	1.000	1.000	1.000	1.000
205	1.450	0.050	0.380	0.844	0.989	1.000	1.000	1.000	1.000	1.000
210	1.448	0.050	0.387	0.852	0.991	1.000	1.000	1.000	1.000	1.000
215	1.447	0.050	0.393	0.859	0.992	1.000	1.000	1.000	1.000	1.000
220	1.446	0.050	0.400	0.867	0.993	1.000	1.000	1.000	1.000	1.000
225	1.444	0.050	0.406	0.874	0.994	1.000	1.000	1.000	1.000	1.000
230	1.443	0.050	0.413	0.880	0.995	1.000	1.000	1.000	1.000	1.000
235	1.442	0.050	0.419	0.887	0.995	1.000	1.000	1.000	1.000	1.000
240	1.440	0.050	0.425	0.893	0.996	1.000	1.000	1.000	1.000	1.000
245	1.439	0.050	0.432	0.899	0.996	1.000	1.000	1.000	1.000	1.000
250	1.438	0.050	0.438	0.904	0.997	1.000	1.000	1.000	1.000	1.000

Table AII.

n	C_0	C_I values 1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30
A. Critical values C_0 for C = 1.50 and powers π at various C_I values for n = 10(5)250 with α = 0.01										
10	2.887	0.010	0.016	0.024	0.035	0.049	0.067	0.090	0.117	0.149
15	2.490	0.010	0.018	0.032	0.052	0.080	0.118	0.166	0.224	0.291
20	2.300	0.010	0.021	0.040	0.071	0.117	0.178	0.256	0.346	0.445
25	2.185	0.010	0.024	0.049	0.092	0.157	0.245	0.352	0.470	0.589
30	2.106	0.010	0.026	0.059	0.115	0.201	0.315	0.447	0.584	0.709
35	2.049	0.010	0.029	0.068	0.140	0.247	0.386	0.538	0.683	0.803
40	2.004	0.010	0.031	0.078	0.165	0.295	0.455	0.621	0.765	0.871
45	1.969	0.010	0.033	0.089	0.192	0.343	0.522	0.694	0.830	0.918
50	1.939	0.010	0.036	0.100	0.219	0.391	0.584	0.757	0.879	0.950
55	1.915	0.010	0.039	0.111	0.247	0.438	0.642	0.809	0.916	0.970
60	1.894	0.010	0.041	0.122	0.275	0.484	0.694	0.852	0.943	0.982
65	1.875	0.010	0.044	0.134	0.304	0.528	0.740	0.887	0.961	0.990
70	1.859	0.010	0.046	0.146	0.333	0.570	0.781	0.914	0.974	0.994
75	1.845	0.010	0.049	0.159	0.361	0.610	0.817	0.936	0.983	0.997
80	1.832	0.010	0.051	0.171	0.390	0.647	0.848	0.952	0.989	0.998
85	1.821	0.010	0.054	0.184	0.418	0.682	0.874	0.965	0.993	0.999
90	1.811	0.010	0.057	0.197	0.446	0.715	0.897	0.974	0.996	0.999
95	1.801	0.010	0.059	0.210	0.473	0.745	0.915	0.981	0.997	1.000
100	1.792	0.010	0.062	0.223	0.499	0.773	0.931	0.987	0.998	1.000
105	1.784	0.010	0.065	0.236	0.525	0.798	0.944	0.990	0.999	1.000
110	1.777	0.010	0.068	0.250	0.551	0.821	0.955	0.993	0.999	1.000
115	1.770	0.010	0.071	0.263	0.575	0.841	0.964	0.995	1.000	1.000
120	1.764	0.010	0.073	0.277	0.599	0.860	0.971	0.997	1.000	1.000
125	1.758	0.010	0.076	0.290	0.622	0.877	0.977	0.998	1.000	1.000
130	1.752	0.010	0.079	0.304	0.644	0.892	0.982	0.998	1.000	1.000
135	1.747	0.010	0.082	0.317	0.665	0.905	0.986	0.999	1.000	1.000
140	1.742	0.010	0.085	0.331	0.685	0.917	0.989	0.999	1.000	1.000
145	1.737	0.010	0.088	0.345	0.705	0.928	0.991	0.999	1.000	1.000
150	1.733	0.010	0.091	0.358	0.723	0.937	0.993	1.000	1.000	1.000
155	1.729	0.010	0.094	0.372	0.741	0.946	0.995	1.000	1.000	1.000
160	1.725	0.010	0.097	0.385	0.757	0.953	0.996	1.000	1.000	1.000
165	1.721	0.010	0.100	0.398	0.773	0.959	0.997	1.000	1.000	1.000
170	1.717	0.010	0.103	0.412	0.788	0.965	0.997	1.000	1.000	1.000
175	1.714	0.010	0.106	0.425	0.802	0.970	0.998	1.000	1.000	1.000
180	1.711	0.010	0.109	0.438	0.816	0.974	0.998	1.000	1.000	1.000
185	1.708	0.010	0.112	0.451	0.829	0.978	0.999	1.000	1.000	1.000
190	1.705	0.010	0.115	0.464	0.841	0.981	0.999	1.000	1.000	1.000
195	1.702	0.010	0.119	0.476	0.852	0.983	0.999	1.000	1.000	1.000
200	1.699	0.010	0.122	0.489	0.862	0.986	0.999	1.000	1.000	1.000
205	1.696	0.010	0.125	0.501	0.872	0.988	1.000	1.000	1.000	1.000
210	1.694	0.010	0.128	0.513	0.882	0.990	1.000	1.000	1.000	1.000
215	1.691	0.010	0.131	0.526	0.890	0.991	1.000	1.000	1.000	1.000
220	1.689	0.010	0.135	0.538	0.899	0.993	1.000	1.000	1.000	1.000
225	1.687	0.010	0.138	0.549	0.906	0.994	1.000	1.000	1.000	1.000
230	1.684	0.010	0.141	0.561	0.913	0.995	1.000	1.000	1.000	1.000
235	1.682	0.010	0.144	0.572	0.920	0.995	1.000	1.000	1.000	1.000
240	1.680	0.010	0.148	0.584	0.926	0.996	1.000	1.000	1.000	1.000
245	1.678	0.010	0.151	0.595	0.932	0.997	1.000	1.000	1.000	1.000
250	1.676	0.010	0.154	0.605	0.937	0.997	1.000	1.000	1.000	1.000

(Continued)

Table AIII.

n	C_0	C_I values 1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30
B. Critical values C_0 *for* C = *1.50 and powers* π *at various* C_I *values for* n = *10(5)250 with* α = *0.025*										
10	2.535	0.025	0.038	0.056	0.079	0.107	0.142	0.183	0.230	0.282
15	2.263	0.025	0.044	0.072	0.111	0.162	0.226	0.299	0.381	0.468
20	2.126	0.025	0.049	0.088	0.145	0.220	0.313	0.417	0.526	0.631
25	2.041	0.025	0.054	0.104	0.179	0.280	0.399	0.527	0.650	0.758
30	1.983	0.025	0.059	0.120	0.215	0.339	0.482	0.624	0.750	0.848
35	1.939	0.025	0.064	0.137	0.250	0.397	0.558	0.708	0.827	0.908
40	1.905	0.025	0.069	0.154	0.286	0.454	0.627	0.776	0.883	0.946
45	1.878	0.025	0.073	0.170	0.322	0.507	0.689	0.831	0.922	0.970
50	1.855	0.025	0.078	0.187	0.357	0.558	0.742	0.874	0.949	0.983
55	1.836	0.025	0.083	0.204	0.392	0.605	0.788	0.908	0.968	0.991
60	1.819	0.025	0.087	0.221	0.426	0.649	0.828	0.933	0.980	0.995
65	1.805	0.025	0.092	0.239	0.459	0.689	0.860	0.952	0.987	0.997
70	1.793	0.025	0.096	0.256	0.491	0.726	0.888	0.966	0.992	0.999
75	1.781	0.025	0.101	0.273	0.522	0.759	0.910	0.976	0.995	0.999
80	1.771	0.025	0.106	0.290	0.552	0.789	0.929	0.983	0.997	1.000
85	1.762	0.025	0.110	0.307	0.580	0.816	0.944	0.988	0.998	1.000
90	1.754	0.025	0.115	0.324	0.608	0.840	0.956	0.992	0.999	1.000
95	1.746	0.025	0.119	0.340	0.634	0.861	0.965	0.994	0.999	1.000
100	1.740	0.025	0.124	0.357	0.659	0.879	0.973	0.996	1.000	1.000
105	1.733	0.025	0.128	0.373	0.682	0.896	0.979	0.997	1.000	1.000
110	1.727	0.025	0.133	0.390	0.705	0.910	0.984	0.998	1.000	1.000
115	1.722	0.025	0.138	0.406	0.726	0.923	0.987	0.999	1.000	1.000
120	1.717	0.025	0.142	0.422	0.746	0.934	0.990	0.999	1.000	1.000
125	1.712	0.025	0.147	0.438	0.765	0.944	0.993	0.999	1.000	1.000
130	1.707	0.025	0.151	0.453	0.782	0.952	0.994	1.000	1.000	1.000
135	1.703	0.025	0.156	0.468	0.799	0.959	0.996	1.000	1.000	1.000
140	1.699	0.025	0.161	0.484	0.814	0.965	0.997	1.000	1.000	1.000
145	1.695	0.025	0.165	0.498	0.829	0.970	0.998	1.000	1.000	1.000
150	1.692	0.025	0.170	0.513	0.842	0.975	0.998	1.000	1.000	1.000
155	1.688	0.025	0.175	0.527	0.855	0.979	0.999	1.000	1.000	1.000
160	1.685	0.025	0.179	0.541	0.866	0.982	0.999	1.000	1.000	1.000
165	1.682	0.025	0.184	0.555	0.877	0.985	0.999	1.000	1.000	1.000
170	1.679	0.025	0.188	0.569	0.887	0.987	0.999	1.000	1.000	1.000
175	1.677	0.025	0.193	0.582	0.896	0.989	1.000	1.000	1.000	1.000
180	1.674	0.025	0.198	0.595	0.905	0.991	1.000	1.000	1.000	1.000
185	1.671	0.025	0.202	0.607	0.913	0.992	1.000	1.000	1.000	1.000
190	1.669	0.025	0.207	0.620	0.920	0.994	1.000	1.000	1.000	1.000
195	1.667	0.025	0.212	0.632	0.927	0.995	1.000	1.000	1.000	1.000
200	1.664	0.025	0.216	0.644	0.934	0.996	1.000	1.000	1.000	1.000
205	1.662	0.025	0.221	0.655	0.939	0.996	1.000	1.000	1.000	1.000
210	1.660	0.025	0.225	0.666	0.945	0.997	1.000	1.000	1.000	1.000
215	1.658	0.025	0.230	0.677	0.949	0.997	1.000	1.000	1.000	1.000
220	1.656	0.025	0.235	0.688	0.954	0.998	1.000	1.000	1.000	1.000
225	1.654	0.025	0.239	0.699	0.958	0.998	1.000	1.000	1.000	1.000
230	1.653	0.025	0.244	0.709	0.962	0.999	1.000	1.000	1.000	1.000
235	1.651	0.025	0.249	0.719	0.965	0.999	1.000	1.000	1.000	1.000
240	1.649	0.025	0.253	0.728	0.968	0.999	1.000	1.000	1.000	1.000
245	1.647	0.025	0.258	0.738	0.971	0.999	1.000	1.000	1.000	1.000
250	1.646	0.025	0.262	0.747	0.974	0.999	1.000	1.000	1.000	1.000

(Continued)

Table AIII.

n	C_0	C_I values 1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30
C. Critical values C_0 for C = 1.50 and powers π at various C_I values for n = 10(5)250 with α = 0.05										
10	2.281	0.050	0.074	0.105	0.143	0.189	0.242	0.300	0.364	0.430
15	2.091	0.050	0.084	0.131	0.192	0.266	0.352	0.444	0.537	0.628
20	1.992	0.050	0.092	0.155	0.239	0.341	0.455	0.570	0.678	0.772
25	1.929	0.050	0.101	0.179	0.285	0.413	0.548	0.675	0.784	0.867
30	1.885	0.050	0.108	0.202	0.330	0.479	0.629	0.760	0.859	0.925
35	1.852	0.050	0.116	0.225	0.374	0.541	0.699	0.825	0.910	0.959
40	1.826	0.050	0.123	0.248	0.416	0.598	0.759	0.875	0.944	0.979
45	1.804	0.050	0.131	0.270	0.456	0.649	0.808	0.911	0.966	0.989
50	1.787	0.050	0.138	0.292	0.494	0.695	0.848	0.938	0.979	0.994
55	1.772	0.050	0.145	0.313	0.531	0.736	0.881	0.957	0.988	0.997
60	1.759	0.050	0.151	0.335	0.566	0.773	0.907	0.971	0.993	0.999
65	1.748	0.050	0.158	0.355	0.598	0.805	0.928	0.980	0.996	0.999
70	1.738	0.050	0.165	0.376	0.629	0.833	0.944	0.987	0.998	1.000
75	1.729	0.050	0.172	0.396	0.658	0.858	0.957	0.991	0.999	1.000
80	1.721	0.050	0.178	0.415	0.685	0.879	0.967	0.994	0.999	1.000
85	1.714	0.050	0.185	0.435	0.711	0.897	0.975	0.996	1.000	1.000
90	1.707	0.050	0.191	0.453	0.735	0.913	0.981	0.997	1.000	1.000
95	1.701	0.050	0.198	0.472	0.757	0.926	0.986	0.998	1.000	1.000
100	1.696	0.050	0.204	0.490	0.777	0.938	0.989	0.999	1.000	1.000
105	1.691	0.050	0.210	0.507	0.796	0.948	0.992	0.999	1.000	1.000
110	1.686	0.050	0.216	0.524	0.814	0.956	0.994	1.000	1.000	1.000
115	1.682	0.050	0.223	0.541	0.830	0.963	0.996	1.000	1.000	1.000
120	1.678	0.050	0.229	0.557	0.845	0.969	0.997	1.000	1.000	1.000
125	1.674	0.050	0.235	0.573	0.859	0.975	0.998	1.000	1.000	1.000
130	1.670	0.050	0.241	0.588	0.871	0.979	0.998	1.000	1.000	1.000
135	1.667	0.050	0.247	0.603	0.883	0.982	0.999	1.000	1.000	1.000
140	1.664	0.050	0.253	0.618	0.894	0.985	0.999	1.000	1.000	1.000
145	1.661	0.050	0.259	0.632	0.904	0.988	0.999	1.000	1.000	1.000
150	1.658	0.050	0.265	0.646	0.913	0.990	0.999	1.000	1.000	1.000
155	1.655	0.050	0.271	0.659	0.921	0.992	1.000	1.000	1.000	1.000
160	1.652	0.050	0.277	0.672	0.928	0.993	1.000	1.000	1.000	1.000
165	1.650	0.050	0.283	0.684	0.935	0.994	1.000	1.000	1.000	1.000
170	1.648	0.050	0.289	0.696	0.941	0.995	1.000	1.000	1.000	1.000
175	1.645	0.050	0.295	0.708	0.947	0.996	1.000	1.000	1.000	1.000
180	1.643	0.050	0.301	0.719	0.952	0.997	1.000	1.000	1.000	1.000
185	1.641	0.050	0.307	0.730	0.957	0.997	1.000	1.000	1.000	1.000
190	1.639	0.050	0.312	0.741	0.961	0.998	1.000	1.000	1.000	1.000
195	1.637	0.050	0.318	0.751	0.965	0.998	1.000	1.000	1.000	1.000
200	1.635	0.050	0.324	0.761	0.968	0.999	1.000	1.000	1.000	1.000
205	1.634	0.050	0.329	0.770	0.971	0.999	1.000	1.000	1.000	1.000
210	1.632	0.050	0.335	0.780	0.974	0.999	1.000	1.000	1.000	1.000
215	1.630	0.050	0.341	0.789	0.977	0.999	1.000	1.000	1.000	1.000
220	1.629	0.050	0.346	0.797	0.979	0.999	1.000	1.000	1.000	1.000
225	1.627	0.050	0.352	0.805	0.981	0.999	1.000	1.000	1.000	1.000
230	1.626	0.050	0.357	0.813	0.983	1.000	1.000	1.000	1.000	1.000
235	1.625	0.050	0.363	0.821	0.985	1.000	1.000	1.000	1.000	1.000
240	1.623	0.050	0.368	0.829	0.987	1.000	1.000	1.000	1.000	1.000
245	1.622	0.050	0.374	0.836	0.988	1.000	1.000	1.000	1.000	1.000
250	1.621	0.050	0.379	0.843	0.989	1.000	1.000	1.000	1.000	1.000

Table AIII.

n	C_0	C_I values 2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80
A. Critical values C_0 for $C = 2.00$ and powers π at various C_I values for $n = 10(5)250$ with $\alpha = 0.01$										
10	3.826	0.010	0.014	0.020	0.027	0.035	0.046	0.059	0.074	0.092
15	3.302	0.010	0.016	0.025	0.037	0.053	0.074	0.101	0.133	0.171
20	3.050	0.010	0.018	0.030	0.048	0.073	0.107	0.150	0.202	0.264
25	2.899	0.010	0.019	0.035	0.060	0.095	0.143	0.205	0.279	0.363
30	2.795	0.010	0.021	0.041	0.072	0.119	0.183	0.264	0.358	0.461
35	2.720	0.010	0.023	0.046	0.085	0.144	0.225	0.324	0.437	0.553
40	2.661	0.010	0.024	0.052	0.099	0.171	0.268	0.385	0.512	0.637
45	2.614	0.010	0.026	0.058	0.113	0.199	0.312	0.445	0.583	0.710
50	2.576	0.010	0.027	0.064	0.128	0.227	0.356	0.503	0.647	0.771
55	2.543	0.010	0.029	0.070	0.144	0.256	0.400	0.557	0.704	0.823
60	2.516	0.010	0.030	0.076	0.159	0.285	0.443	0.609	0.755	0.864
65	2.492	0.010	0.032	0.082	0.176	0.315	0.485	0.656	0.799	0.897
70	2.471	0.010	0.033	0.089	0.192	0.345	0.526	0.700	0.836	0.923
75	2.452	0.010	0.035	0.095	0.209	0.374	0.565	0.739	0.867	0.943
80	2.435	0.010	0.036	0.102	0.226	0.404	0.602	0.775	0.893	0.958
85	2.420	0.010	0.038	0.109	0.243	0.432	0.637	0.806	0.915	0.970
90	2.407	0.010	0.039	0.116	0.260	0.461	0.670	0.834	0.933	0.978
95	2.394	0.010	0.041	0.123	0.277	0.489	0.701	0.859	0.947	0.984
100	2.383	0.010	0.043	0.130	0.295	0.516	0.729	0.880	0.959	0.989
105	2.372	0.010	0.044	0.137	0.312	0.542	0.756	0.899	0.968	0.992
110	2.363	0.010	0.046	0.145	0.330	0.568	0.781	0.915	0.975	0.994
115	2.354	0.010	0.047	0.152	0.348	0.593	0.803	0.928	0.981	0.996
120	2.345	0.010	0.049	0.160	0.365	0.617	0.824	0.940	0.985	0.997
125	2.337	0.010	0.050	0.167	0.382	0.640	0.843	0.950	0.989	0.998
130	2.330	0.010	0.052	0.175	0.400	0.662	0.860	0.959	0.991	0.999
135	2.323	0.010	0.054	0.183	0.417	0.683	0.875	0.966	0.993	0.999
140	2.317	0.010	0.055	0.190	0.434	0.703	0.889	0.972	0.995	0.999
145	2.311	0.010	0.057	0.198	0.450	0.722	0.902	0.977	0.996	1.000
150	2.305	0.010	0.059	0.206	0.467	0.740	0.913	0.981	0.997	1.000
155	2.299	0.010	0.060	0.214	0.483	0.758	0.924	0.984	0.998	1.000
160	2.294	0.010	0.062	0.222	0.499	0.774	0.933	0.987	0.998	1.000
165	2.289	0.010	0.063	0.230	0.515	0.789	0.941	0.990	0.999	1.000
170	2.284	0.010	0.065	0.238	0.531	0.804	0.948	0.991	0.999	1.000
175	2.280	0.010	0.067	0.246	0.546	0.818	0.954	0.993	0.999	1.000
180	2.276	0.010	0.068	0.254	0.561	0.831	0.960	0.994	1.000	1.000
185	2.272	0.010	0.070	0.263	0.576	0.843	0.965	0.995	1.000	1.000
190	2.268	0.010	0.072	0.271	0.590	0.855	0.970	0.996	1.000	1.000
195	2.264	0.010	0.074	0.279	0.604	0.865	0.973	0.997	1.000	1.000
200	2.260	0.010	0.075	0.287	0.618	0.875	0.977	0.998	1.000	1.000
205	2.257	0.010	0.077	0.295	0.632	0.885	0.980	0.998	1.000	1.000
210	2.253	0.010	0.079	0.303	0.645	0.894	0.983	0.998	1.000	1.000
215	2.250	0.010	0.080	0.312	0.658	0.902	0.985	0.999	1.000	1.000
220	2.247	0.010	0.082	0.320	0.670	0.910	0.987	0.999	1.000	1.000
225	2.244	0.010	0.084	0.328	0.683	0.917	0.989	0.999	1.000	1.000
230	2.241	0.010	0.086	0.336	0.694	0.923	0.990	0.999	1.000	1.000
235	2.238	0.010	0.088	0.344	0.706	0.929	0.992	1.000	1.000	1.000
240	2.236	0.010	0.089	0.352	0.717	0.935	0.993	1.000	1.000	1.000
245	2.233	0.010	0.091	0.361	0.728	0.940	0.994	1.000	1.000	1.000
250	2.230	0.010	0.093	0.369	0.739	0.945	0.995	1.000	1.000	1.000

(Continued)

Table AIV.

n	C_0	C_I values 2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80
B. Critical values C_0 for C = 2.00 and powers π at various C_I values for n = 10(5)250 with α = 0.025										
10	3.361	0.025	0.035	0.047	0.062	0.080	0.101	0.126	0.155	0.187
15	3.002	0.025	0.039	0.057	0.082	0.113	0.152	0.197	0.249	0.307
20	2.821	0.025	0.042	0.067	0.102	0.148	0.205	0.271	0.346	0.427
25	2.710	0.025	0.046	0.078	0.123	0.184	0.259	0.346	0.442	0.539
30	2.633	0.025	0.049	0.088	0.144	0.220	0.314	0.420	0.531	0.638
35	2.575	0.025	0.052	0.098	0.166	0.257	0.368	0.490	0.611	0.721
40	2.531	0.025	0.055	0.108	0.187	0.294	0.421	0.555	0.682	0.789
45	2.495	0.025	0.058	0.118	0.209	0.331	0.472	0.615	0.743	0.843
50	2.465	0.025	0.061	0.128	0.231	0.367	0.521	0.669	0.794	0.884
55	2.440	0.025	0.064	0.138	0.253	0.403	0.567	0.718	0.837	0.916
60	2.418	0.025	0.067	0.148	0.275	0.438	0.610	0.761	0.872	0.940
65	2.399	0.025	0.070	0.158	0.297	0.472	0.650	0.798	0.900	0.957
70	2.383	0.025	0.073	0.169	0.319	0.504	0.687	0.831	0.922	0.970
75	2.368	0.025	0.076	0.179	0.340	0.536	0.721	0.859	0.940	0.979
80	2.355	0.025	0.078	0.189	0.362	0.566	0.752	0.883	0.954	0.986
85	2.343	0.025	0.081	0.200	0.383	0.595	0.780	0.903	0.965	0.990
90	2.332	0.025	0.084	0.210	0.404	0.623	0.806	0.920	0.974	0.993
95	2.323	0.025	0.087	0.220	0.425	0.649	0.829	0.934	0.980	0.995
100	2.313	0.025	0.090	0.231	0.445	0.674	0.849	0.946	0.985	0.997
105	2.305	0.025	0.092	0.241	0.465	0.698	0.868	0.956	0.989	0.998
110	2.297	0.025	0.095	0.251	0.484	0.720	0.884	0.964	0.992	0.999
115	2.290	0.025	0.098	0.262	0.503	0.741	0.899	0.971	0.994	0.999
120	2.283	0.025	0.101	0.272	0.522	0.761	0.912	0.977	0.996	0.999
125	2.277	0.025	0.103	0.282	0.540	0.779	0.923	0.981	0.997	1.000
130	2.271	0.025	0.106	0.293	0.558	0.796	0.934	0.985	0.998	1.000
135	2.266	0.025	0.109	0.303	0.575	0.812	0.942	0.988	0.998	1.000
140	2.261	0.025	0.112	0.313	0.592	0.827	0.950	0.990	0.999	1.000
145	2.256	0.025	0.114	0.323	0.609	0.841	0.957	0.992	0.999	1.000
150	2.251	0.025	0.117	0.333	0.625	0.854	0.963	0.994	0.999	1.000
155	2.247	0.025	0.120	0.343	0.640	0.867	0.968	0.995	1.000	1.000
160	2.242	0.025	0.123	0.353	0.655	0.878	0.973	0.996	1.000	1.000
165	2.238	0.025	0.125	0.363	0.670	0.888	0.976	0.997	1.000	1.000
170	2.235	0.025	0.128	0.373	0.684	0.898	0.980	0.998	1.000	1.000
175	2.231	0.025	0.131	0.383	0.697	0.907	0.983	0.998	1.000	1.000
180	2.227	0.025	0.134	0.393	0.710	0.915	0.985	0.999	1.000	1.000
185	2.224	0.025	0.136	0.403	0.723	0.922	0.987	0.999	1.000	1.000
190	2.221	0.025	0.139	0.412	0.735	0.929	0.989	0.999	1.000	1.000
195	2.218	0.025	0.142	0.422	0.747	0.935	0.991	0.999	1.000	1.000
200	2.215	0.025	0.145	0.431	0.759	0.941	0.992	0.999	1.000	1.000
205	2.212	0.025	0.147	0.441	0.770	0.947	0.993	1.000	1.000	1.000
210	2.209	0.025	0.150	0.450	0.780	0.951	0.994	1.000	1.000	1.000
215	2.207	0.025	0.153	0.459	0.790	0.956	0.995	1.000	1.000	1.000
220	2.204	0.025	0.156	0.469	0.800	0.960	0.996	1.000	1.000	1.000
225	2.202	0.025	0.158	0.478	0.809	0.964	0.997	1.000	1.000	1.000
230	2.199	0.025	0.161	0.487	0.818	0.967	0.997	1.000	1.000	1.000
235	2.197	0.025	0.164	0.495	0.827	0.970	0.998	1.000	1.000	1.000
240	2.195	0.025	0.167	0.504	0.835	0.973	0.998	1.000	1.000	1.000
245	2.193	0.025	0.169	0.513	0.843	0.976	0.998	1.000	1.000	1.000
250	2.191	0.025	0.172	0.522	0.851	0.978	0.999	1.000	1.000	1.000

(Continued)

Table AIV.

n	C_0	2.00	2.10	2.20	C_I values 2.30	2.40	2.50	2.60	2.70	2.80
C. Critical values C_0 for C = 2.00 and powers π at various C_I values for n = 10(5)250 with α = 0.05										
10	3.026	0.050	0.068	0.089	0.115	0.145	0.179	0.218	0.260	0.306
15	2.776	0.050	0.074	0.106	0.147	0.195	0.251	0.314	0.382	0.452
20	2.645	0.050	0.081	0.123	0.177	0.244	0.321	0.405	0.493	0.581
25	2.562	0.050	0.086	0.138	0.207	0.291	0.388	0.490	0.592	0.687
30	2.504	0.050	0.092	0.153	0.236	0.338	0.451	0.567	0.676	0.771
35	2.461	0.050	0.097	0.168	0.265	0.382	0.510	0.635	0.746	0.835
40	2.426	0.050	0.102	0.183	0.293	0.425	0.565	0.695	0.803	0.884
45	2.399	0.050	0.106	0.197	0.321	0.467	0.615	0.747	0.849	0.919
50	2.376	0.050	0.111	0.211	0.348	0.506	0.661	0.791	0.885	0.944
55	2.356	0.050	0.116	0.225	0.374	0.543	0.702	0.829	0.914	0.962
60	2.339	0.050	0.120	0.239	0.400	0.578	0.740	0.860	0.935	0.974
65	2.324	0.050	0.125	0.252	0.425	0.611	0.773	0.887	0.952	0.983
70	2.311	0.050	0.129	0.266	0.449	0.642	0.802	0.908	0.964	0.989
75	2.300	0.050	0.133	0.279	0.473	0.672	0.829	0.926	0.974	0.992
80	2.289	0.050	0.138	0.292	0.496	0.699	0.852	0.941	0.981	0.995
85	2.280	0.050	0.142	0.305	0.519	0.724	0.872	0.953	0.986	0.997
90	2.271	0.050	0.146	0.318	0.540	0.748	0.890	0.962	0.990	0.998
95	2.264	0.050	0.150	0.331	0.561	0.769	0.905	0.970	0.993	0.999
100	2.256	0.050	0.154	0.344	0.581	0.790	0.919	0.976	0.995	0.999
105	2.250	0.050	0.158	0.356	0.601	0.808	0.931	0.981	0.996	0.999
110	2.243	0.050	0.162	0.369	0.620	0.826	0.941	0.985	0.997	1.000
115	2.238	0.050	0.166	0.381	0.638	0.841	0.950	0.988	0.998	1.000
120	2.232	0.050	0.170	0.393	0.655	0.856	0.957	0.991	0.999	1.000
125	2.227	0.050	0.174	0.405	0.672	0.869	0.963	0.993	0.999	1.000
130	2.223	0.050	0.178	0.416	0.688	0.882	0.969	0.995	0.999	1.000
135	2.218	0.050	0.182	0.428	0.704	0.893	0.974	0.996	1.000	1.000
140	2.214	0.050	0.186	0.439	0.718	0.903	0.978	0.997	1.000	1.000
145	2.210	0.050	0.190	0.451	0.732	0.912	0.981	0.997	1.000	1.000
150	2.206	0.050	0.194	0.462	0.746	0.921	0.984	0.998	1.000	1.000
155	2.203	0.050	0.197	0.473	0.759	0.929	0.987	0.998	1.000	1.000
160	2.199	0.050	0.201	0.484	0.771	0.936	0.989	0.999	1.000	1.000
165	2.196	0.050	0.205	0.494	0.783	0.942	0.991	0.999	1.000	1.000
170	2.193	0.050	0.209	0.505	0.795	0.948	0.992	0.999	1.000	1.000
175	2.190	0.050	0.213	0.515	0.805	0.953	0.993	0.999	1.000	1.000
180	2.187	0.050	0.216	0.525	0.816	0.958	0.994	1.000	1.000	1.000
185	2.185	0.050	0.220	0.535	0.826	0.962	0.995	1.000	1.000	1.000
190	2.182	0.050	0.224	0.545	0.835	0.966	0.996	1.000	1.000	1.000
195	2.180	0.050	0.228	0.555	0.844	0.969	0.997	1.000	1.000	1.000
200	2.177	0.050	0.231	0.564	0.852	0.973	0.997	1.000	1.000	1.000
205	2.175	0.050	0.235	0.574	0.860	0.975	0.998	1.000	1.000	1.000
210	2.173	0.050	0.239	0.583	0.868	0.978	0.998	1.000	1.000	1.000
215	2.171	0.050	0.242	0.592	0.875	0.980	0.998	1.000	1.000	1.000
220	2.169	0.050	0.246	0.601	0.882	0.982	0.999	1.000	1.000	1.000
225	2.167	0.050	0.250	0.610	0.889	0.984	0.999	1.000	1.000	1.000
230	2.165	0.050	0.253	0.618	0.895	0.986	0.999	1.000	1.000	1.000
235	2.163	0.050	0.257	0.627	0.901	0.987	0.999	1.000	1.000	1.000
240	2.161	0.050	0.260	0.635	0.907	0.989	0.999	1.000	1.000	1.000
245	2.159	0.050	0.264	0.643	0.912	0.990	0.999	1.000	1.000	1.000
250	2.158	0.050	0.268	0.651	0.917	0.991	1.000	1.000	1.000	1.000

Table AIV.